

The photon shuttle: Landau-Zener-Stueckelberg dynamics in an optomechanical system

Georg Heinrich,¹ J. G. E. Harris,^{2,3} and Florian Marquardt¹

¹*Arnold Sommerfeld Center for Theoretical Physics,
Center for NanoScience and Department of Physics,*

Ludwig-Maximilians-Universität München, Theresienstr. 37, D-80333 München, Germany

²*Department of Physics, Yale University, 217 Prospect Street, New Haven, Connecticut, 06520, USA*

³*Department of Applied Physics, Yale University,
15 Prospect Street, New Haven, Connecticut 06520, USA*

The motion of micro- and nanomechanical resonators can be coupled to electromagnetic fields. Such optomechanical setups allow one to explore the interaction of light and matter in a new regime at the boundary between quantum and classical physics. We propose an approach to investigate non-equilibrium photon dynamics driven by mechanical motion in a recently developed setup with a membrane between two mirrors, where photons can be shuttled between the two halves of the cavity. For modest driving strength we predict the possibility to observe an Autler-Townes splitting indicative of Rabi dynamics. For large drive, we show that this system displays Landau-Zener-Stueckelberg dynamics originally known from atomic two-state systems.

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Landau-Zener (LZ) transitions [1, 2] are essential to the dynamics of many physical systems. In the usual model, a parameter in a two-state Hamiltonian is swept through an avoided level crossing where the two bare eigenstates $|1\rangle$ and $|2\rangle$ hybridize. When the parameter is changed at a finite speed, the system may undergo a LZ transition into the other eigenstate. Beyond this standard LZ problem, the dynamics becomes more elaborate if repeated transitions are taken into account. For a periodic modulation of the parameter, the first LZ transition splits the state into a coherent superposition $\alpha|1\rangle + \beta|2\rangle$. Due to the difference in energy, the system afterwards accumulates a relative phase between states $|1\rangle$ and $|2\rangle$. Thus, when returned to the avoided crossing, the system undergoes quantum interference with itself during the second LZ transition. This leads to interference patterns for the state population, so called Stueckelberg oscillations [3]. Originally, Landau-Zener-Stueckelberg (LZS) dynamics was studied in atomic systems [4, 5, 6]. Recently, the concept has been applied to superconducting qubits [7]. Currently, there is growing interest in LZ and LZS dynamics concerning topics such as state preparation and entanglement [8, 9], cooling or qubit spectroscopy [10]. Another rapidly evolving area of research is optomechanics (see [11] for a recent review and further references). Optomechanical systems couple mechanical degrees of freedom to radiation fields. This provides new means to manipulate both the light field and the mechanical motion. Apart from the hope to eventually explore the quantum regime of mechanical motion, there have been several studies of the complex nonlinear dynamics of these systems [12, 13, 14, 15].

Here, we propose an approach to observe dynamics of the light field in a driven optomechanical system, in the form of LZS oscillations. We note that there exist some

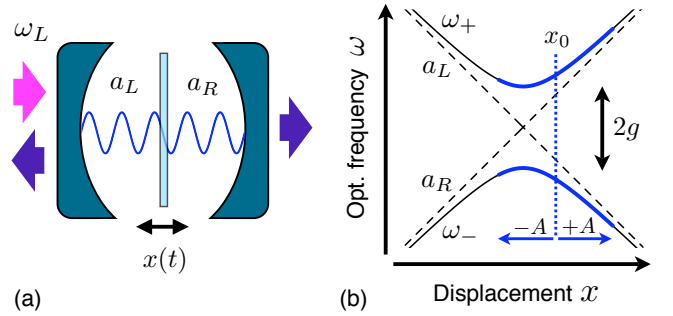


Figure 1: (a) Setup: a dielectric membrane couples two modes a_L, a_R inside a cavity. The left hand side is excited by a laser ω_L while the transmission to the right is recorded. (b) Optical resonance frequency as function of displacement: the membrane's displacement linearly changes the bare mode frequencies (dashed). Due to the coupling g , there is an avoided crossing of the eigenfrequencies ω_{\pm} (black). The membrane is driven, with $x(t) = A \cos(\Omega t) + x_0$ (blue).

purely optical setups [16, 17] that have mimicked quantum two-state and standard LZ dynamics (but not LZS oscillations). In the optomechanical setup analyzed here, the mechanical motion of a membrane placed between two fixed mirrors is driven such that the resulting motion shuttles photons between the two halves of the cavity. A setup of this kind was recently realized in [18, 19].

We consider two cavity modes coupled by a dielectric membrane placed in the middle between two high-finesse mirrors, see Fig. 1a. The system Hamiltonian reads

$$\begin{aligned} \hat{H}_{\text{sys}} = & \hbar\omega_0 \left(1 - \frac{x(t)}{l}\right) \hat{a}_L^\dagger \hat{a}_L + \hbar\omega_0 \left(1 + \frac{x(t)}{l}\right) \hat{a}_R^\dagger \hat{a}_R \\ & + \hbar g \left(\hat{a}_L^\dagger \hat{a}_R + \hat{a}_R^\dagger \hat{a}_L\right) + \hat{H}_{\text{drive}} + \hat{H}_{\text{decay}}. \end{aligned} \quad (1)$$

$\hat{a}_L^\dagger \hat{a}_L$ and $\hat{a}_R^\dagger \hat{a}_R$ are the photon numbers for the two optical modes in the left and right cavity half (each of length l), respectively, whose resonance frequency ω_0 is changed due to the displacement x of the membrane. The coupling g describes the photon tunneling through the membrane. Due to the coupling, there is an avoided crossing in the optical resonance frequency $\omega_\pm(x) = \pm\sqrt{g^2 + (\omega_0 x/l)^2}$, see Fig. 1b. We propose to drive the membrane with mechanical frequency Ω and resulting amplitude A around a mean position x_0 ,

$$x(t) = A \cos(\Omega t) + x_0, \quad (2)$$

and investigate the system in the regime where the timescale of photon exchange is comparable to the timescale of the mechanical motion ($g \simeq \Omega$). Recently the coupling frequency $g/2\pi$ has been significantly reduced by exploiting properties of transverse modes [20], and it is tunable down to 200 kHz at present. The mechanical eigenfrequencies of typical 1 mm \times 1 mm \times 50 nm membranes range between 100 kHz and 1 MHz. Commercially available membrane sizes should allow this to go from 20 kHz up to 10 MHz. We point out that here Ω need not coincide with the membrane's eigenfrequency but depends only on the driving.

We assume the left hand side of the cavity is driven by a laser of frequency ω_L and amplitude b_L^{in} . Our goal is to examine the photon dynamics by looking at the transmission T . Using input/output theory, the equations of motion for the average fields $a_L = \langle \hat{a}_L \rangle$ and $a_R = \langle \hat{a}_R \rangle$ read

$$\begin{aligned} \frac{d}{dt} a_L &= \frac{1}{i} [-\bar{x}(t) a_L + g a_R] - \frac{\kappa}{2} a_L - \sqrt{\kappa} b_L^{in}(t) \\ \frac{d}{dt} a_R &= \frac{1}{i} [\bar{x}(t) a_R + g a_L] - \frac{\kappa}{2} a_R, \end{aligned} \quad (3)$$

with the cavity decay rate κ for each of the modes, and the drive $b_L^{in}(t) = e^{-i\Delta_L t} b_L^{in}$. Here, we used a rotating frame, with laser detuning from resonance $\Delta_L = \omega_L - \omega_0$. The displacement is written in terms of a frequency via $\bar{x}(t) = (\omega_0/l)x(t)$, likewise for \bar{A} , \bar{x}_0 . The transmission to the right, $T(t) = \kappa \langle \hat{a}_R^\dagger(t) \hat{a}_R(t) \rangle / (b_L^{in})^2$, can be expressed as

$$T(t) = \kappa^2 \left| \int_{-\infty}^t G(t, t') e^{-i\Delta_L t' - (\kappa/2)(t-t')} dt' \right|^2, \quad (4)$$

where the phase factor includes laser drive and cavity decay, while the Green's function $G(t, t')$ describes the amplitude for a photon to enter the cavity from the left at time t' and to be found in the right cavity mode later at time t . Technically, $G(t, t')$ is found by setting $\kappa = 0$ in Eq. (3) and solving for $a_R(t)$ with the initial conditions $a_L(t') = 1$, $a_R(t') = 0$.

We start investigating the dynamics by considering modest drive amplitudes $\bar{A} < \Omega$. Fig. 2a displays the time-averaged transmission depending on \bar{x}_0 and Δ_L . We

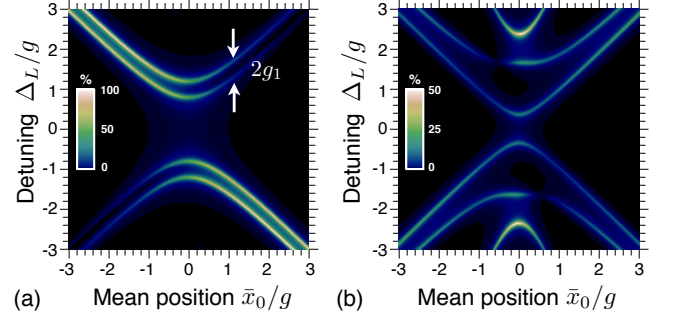


Figure 2: (a) Autler-Townes splitting of the cavity frequencies ω_\pm due to the mechanical motion: density plot for the time-averaged transmission depending on \bar{x}_0 and Δ_L . For every position \bar{x}_0 the mechanical drive frequency is set to be $\Omega = 2\sqrt{g^2 + \bar{x}_0^2}$ such that it is always resonant with the difference between the two optical mode frequencies ω_\pm . Further parameters are amplitude $\bar{A} = 0.2\Omega$ and decay $\kappa = 0.1g$. The splitting is set by the Rabi frequency $g_1 \simeq g\bar{A}/\Omega$, proportional to the drive amplitude. (b) Transmission as in (a) but for stronger drive $\bar{A} = 1.6\Omega$. Mechanical sidebands, displaced by $\pm\Omega$, become visible and interact.

observe an Autler-Townes splitting [16, 21] of the two hyperbola branches ω_\pm . Indeed, the mechanical drive induces Rabi oscillations between the two photon branches, at a Rabi frequency $g_1 \simeq g\bar{A}/\Omega$, leading to a corresponding splitting in the spectroscopic picture. For larger drive amplitudes the dynamics becomes more involved. For instance mechanical sidebands arise as shown in Fig. 2b, and they start to interact with each other. In the following, we will focus on the dynamics of this strong driving regime.

Fig. 3 shows numerical results for $\bar{A} \gg \Omega$, g . For experimentally accessible parameters $g/2\pi = 1$ MHz, $l = 1$ cm and $\omega_0/2\pi = 3 \cdot 10^{14}$ Hz, we have $\omega_0/l = 30g/\text{nm}$ and $\bar{A} = 60g$ corresponds to an oscillation amplitude of $A = 2$ nm that is below the nonlinear regime for a 50 nm thick membrane. Apart from the modulation of transmission as a function of \bar{A} (see below), we observe finite transmission only if \bar{x}_0 is a multiple of Ω . We first present an intuitive description. Transmission is determined by two subsequent processes. First, the laser has to excite the left mode. Secondly, the internal dynamics must be able to transfer photons into the right one. In general, both processes are inelastic and therefore require energy to be transferred between the light field and the oscillating membrane. The left mode's frequency is oscillating around the time-averaged value $-\bar{x}_0$. Hence, the resonance condition to excite the left mode reads

$$\Delta_L + m\Omega = -\bar{x}_0, \quad (5)$$

see Fig. 4a. Here, $m\Omega$ is an adequate multiphonon transition. The width of the individual resonances is determined by κ . The subsequent process displays the physics of LZS dynamics: LZ transitions split the photon state

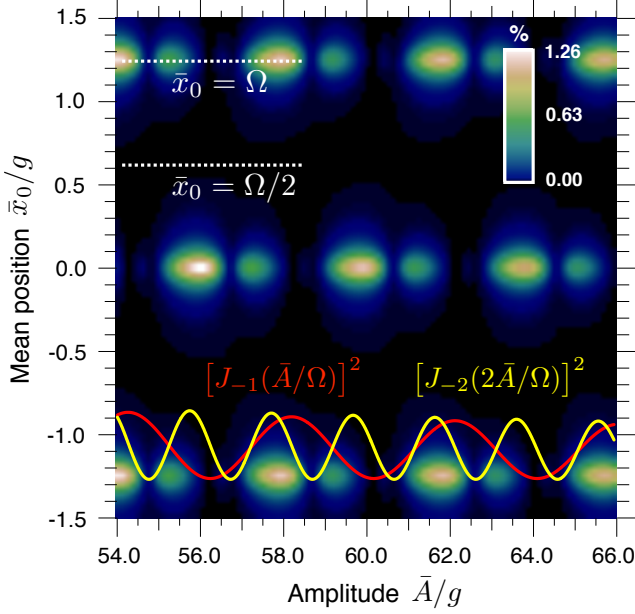


Figure 3: Density plot for the time-averaged transmission as function of average displacement \bar{x}_0 and mechanical drive amplitude $\bar{A} \gg \Omega, g$. Further parameters are laser detuning $\Delta_L = 0$, mechanical frequency $\Omega/2\pi = 0.2g$ and cavity decay $\kappa = 0.2g$. Finite transmission is observed when the resonance conditions Eq. (5) and (6) for multiphonon transitions are met. The transmission is modulated according to the product of two Bessel functions. For the case $\bar{x}_0 = -\Omega$ both are depicted in the plot's plane. Red: $\sim J_m^2(\bar{A}/\Omega)$, due to the excitation process. Yellow: $\sim J_n^2(2\bar{A}/\Omega)$, due to LZS dynamics.

into a coherent superposition, the two amplitudes gather different phases and interfere the next time the system transverses the avoided crossing. The condition for constructive interference can also be phrased in terms of an additional multiphonon transition that transfers a photon from the left mode with average frequency $-\bar{x}_0$ to the right one at $+\bar{x}_0$,

$$n\Omega = 2\bar{x}_0. \quad (6)$$

We find transmission only if both conditions are met. We note that the coupling g between modes does not enter here. We will come back to this point later.

To derive these resonance conditions as well as to understand the dependence on \bar{A} , in the following, we calculate an approximate, analytic expression for the transmission. From Eq. (3), the Green's function $G(t, t')$, required for the transmission (4), is found to be

$$G(t, t') = \tilde{a}_R(t, t')e^{-i\phi(t')}, \quad (7)$$

where we have split off a phase $\phi(t') = (\bar{A}/\Omega)\sin(\Omega t')$, and $\tilde{a}_R(t, t')$ is a solution to

$$i\frac{d}{dt} \begin{pmatrix} \tilde{a}_R \\ \tilde{a}_L \end{pmatrix} = \begin{pmatrix} \bar{x}_0 & ge^{+2i\phi(t)} \\ ge^{-2i\phi(t)} & -\bar{x}_0 \end{pmatrix} \begin{pmatrix} \tilde{a}_R \\ \tilde{a}_L \end{pmatrix}, \quad (8)$$

with $t \geq t'$ and initial condition $\tilde{a}_R(t', t') = 0, \tilde{a}_L(t', t') = 1$. We now show that the two multiphonon processes introduced above correspond to the two factors in Eq. (7). The term $e^{-i\phi(t')} = \sum_m J_m(\bar{A}/\Omega)e^{-im\Omega t'}$ describes the initial excitation, where the amplitude for a transfer of m phonons is set by the Bessel function $J_m(\bar{A}/\Omega)$. Secondly, the internal dynamics described by $\tilde{a}_R(t, t')$ is expressed in terms of a two-level system with time-dependent coupling $ge^{2i\phi(t)} = g \sum_n J_n(2\bar{A}/\Omega)e^{in\Omega t}$. Thus, the strength of the second multiphonon transition $n\Omega$ in Fig. 4a is determined by a Bessel function $J_n(2\bar{A}/\Omega)$, corresponding to Stueckelberg interferences known from atomic physics. As a special case, this also describes the Autler-Townes splitting at small drive. This can be calculated from (8) using an interaction picture representation and considering the time-dependent coupling only up to J_1 , yielding an effective transition frequency $2\sqrt{g_0^2 + \bar{x}_0^2}$, with $g_0 = gJ_0(2\bar{A}/\Omega)$, and a Rabi frequency $g_1 = gJ_1(2\bar{A}/\Omega)$.

In the case of LZS dynamics, i.e. strong drive, for sufficiently large amplitudes only one of the harmonics of $g \sum_n J_n(2\bar{A}/\Omega)e^{in\Omega t}$ will be in resonance with the system. This corresponds to leading-order perturbation theory within the Floquet approach [22] applied to Eq. (8). In this case Eq. (8) simplifies to the problem of a two-state system with harmonic drive at $n\Omega$ and effective coupling constant

$$g_n = gJ_n(2\bar{A}/\Omega). \quad (9)$$

To estimate when this approximation becomes appropriate, we note that for a driven undamped two-state system the width of the power-broadened resonance is set by the Rabi frequency. Thus, Eq. (8) yields a series of resonance peaks at $\bar{x}_0 = n\Omega/2$, and they become separated if $4g_n < \Omega$. Using the asymptotic form for large arguments $\bar{A}/\Omega \gg 1$, $J_n(y) \simeq \sqrt{\frac{2}{\pi y}} \cos(y - \frac{n\pi}{2} - \frac{\pi}{4})$, we find the resonance approximation to hold whenever

$$g^2 < \frac{\pi}{16} \bar{A}\Omega. \quad (10)$$

Note the resemblance to the criterion for non-adiabatic transitions that can be derived from the standard LZ formula $P_{1 \rightarrow 1} = \exp(-\pi g^2/2v)$, where $v = \bar{A}\Omega$ is the sweep velocity. Eq. (10) is clearly fulfilled for the parameters of Fig. 3.

Given the resonance approximation, we find for the Green's function

$$G(t, t') = -i \frac{g_n}{\omega_n} \sin(\omega_n(t - t')) e^{-in\Omega(t+t')/2} e^{-i\phi(t')}, \quad (11)$$

with $\omega_n = \sqrt{(g_n)^2 + (\bar{x}_0 - n\Omega/2)^2}$. Note that ω_n contains g_n , which is much smaller than the bare splitting g for $\bar{A} \gg \Omega$. This explains why the resonance conditions (5) and (6) involve the bare optical mode frequencies $\pm\bar{x}_0$ instead of the adiabatic eigenfrequencies ω_{\pm} .

We insert (11) into (4), taking into account the sum over independent contributions with n quanta. In the resolved sideband regime ($\Omega > \kappa$), the integration of (4) selects a specific m for the excitation process, see Eq. (5). Thus we find an approximate expression for the transmission (displayed here for the special case $\Delta_L = 0$, where $m = 2n$):

$$T = \left(\frac{\kappa}{g}\right)^2 \sum_n \left(J_n\left(\frac{\bar{A}}{\Omega}\right) \times \frac{J_{2n}\left(2\frac{\bar{A}}{\Omega}\right)}{\frac{1}{g^2} \left[\left(\frac{\kappa}{2}\right)^2 + (x_0 - n\Omega)^2 \right] + \left[J_{2n}\left(2\frac{\bar{A}}{\Omega}\right) \right]^2} \right)^2 \quad (12)$$

This captures fully the numerical results shown in Fig. 3. Whenever the resonance conditions are fulfilled (by choosing the offset \bar{x}_0), the transmission is modulated by the two Bessel functions. While $J_m(\bar{A}/\Omega)$ describing the excitation process depends on the amplitude \bar{A} , the LZS dynamics characterized by $J_n(2\bar{A}/\Omega)$ is determined by the phase difference gathered between LZ transitions, involving $2\bar{A}$. According to Eq. (5), if we were to increase Δ_L in Fig. 3 we would tune out of resonance and the transmission would vanish everywhere. For $\Delta_L = \Omega/2$, the conditions (5) and (6) can be met for \bar{x}_0 being an *odd* multiple of $\Omega/2$. For $\Delta_L = \Omega$ we would again find transmission for \bar{x}_0 being a multiple of Ω . Note however that the entire plot would be shifted in \bar{A} by an amount $\pi\Omega/2$ due to the changed index of the Bessel function J_m . Finally, at $\Delta_L = 2\Omega$ we would recover Fig. 3.

Fig. 4b shows numerical results for smaller values of \bar{A}/g (while keeping Ω as in Fig. 3). As before, we see resonances for \bar{x}_0 being a multiple of Ω and expect regions of excitation with width κ determined by $J_m(\bar{A}/\Omega)$. Within these regions, we note the already familiar substructure that is due to LZS dynamics.

To conclude, we proposed a setup to investigate non-equilibrium photon dynamics driven by mechanical motion in an optomechanical system with a membrane inside a cavity. We predicted the possibility to observe Autler-Townes splitting and features of Landau-Zener-Stueckelberg dynamics in the transmission spectrum. The observation of the effects discussed here is within reach of current experiments. The same nontrivial light field dynamics will enter when describing self-induced nonlinear optomechanical oscillations in these systems, which would be an interesting topic for future research.

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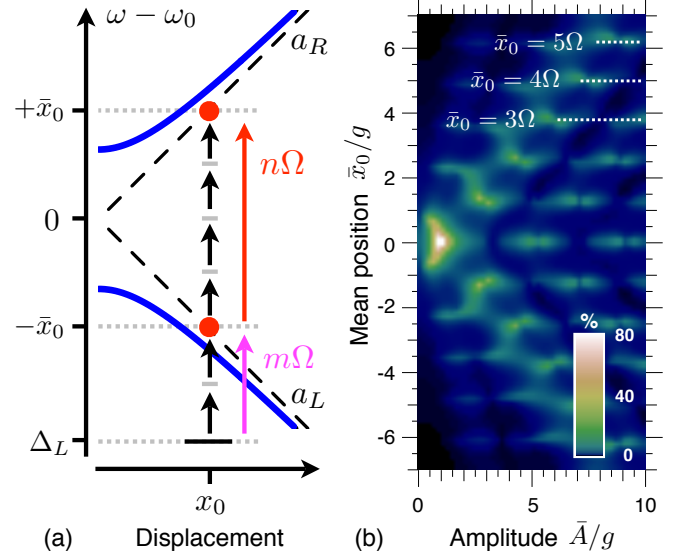


Figure 4: (a) Multiphonon transition picture. To see transmission, two processes are involved: Firstly (magenta), excitation of the left cavity mode by the laser drive at Δ_L , supported by m phonons. Secondly (red), a suitable n -quanta multiphonon transition to transfer a photon from the left into the right mode. (b) Density plot for time-averaged transmission as a function of \bar{x}_0 and $\bar{A} \simeq g$. Further parameters as in Fig. 3.

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